Reasoning Algebraically About An Operation Grades 3-5

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The Progression in Operations and Algebraic Thinking deals with the basic operations—the kinds of quantitative relationships they model and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships.

Although most of the standards organized under the OA heading involve whole numbers, the importance of the Progression is much more general because it describes concepts, properties, and representations that extend to other number systems, to measures, and to algebra.

Solve the following tasks with representations and equations:

Candies are packaged in a box that has 7 rows and 8 columns. How many candies are in a box?

Compare these two….

A case of candies has 7 rows and 8 columns in each layer and it has 10 layers. How many candies are in a case?

A case of candies has 10 times more rows than a box and the same number of columns. How many candies are in a case?

Candies in a Box and in a Case

What representations did you have for these two tasks?

What equations could represent the box and the case?

What connections do you see between the representations and the equations?

Candies in a Box and in a Case

Box: 7 x 8 or 8 x 7

Case: 10 x 7 x 8 or… 10 x (7 x 8)

OR (10 x 7) x 8 or…. (10 x 8) x 7 Or 56 x 10

When we think about how students may calculate the number of candies in a case how might students connect the representations to the expressions?

What understandings about place value and the properties of multiplication might arise as students discuss/ solve these expressions?

Standards?

Which standards apply to this work?

3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

3.OA.5 Apply properties of operations as strategies to multiply and divide.2 *Examples: If 6 × 4 = 24 is known, then 4 × 6 = 24 is also known. (Commutative property of multiplication.) 3 × 5 × 2 can be found by 3 × 5 = 15, then 15 × 2 = 30, or by 5 × 2 = 10, then 3 × 10 = 30.*

Solve the following task with representations and equations:

There are 8 flowers in the Ms. Sanchez' flower pot. There are 25 times more flowers in the class garden. How many flowers are in the garden?

There are 16 flowers in Ms. Sanchez' flower pot. There are 25 times more flowers in the class garden. How many flowers are in the garden?

Representations for the Flower Task

Equations for the Flower Task?

Grade 4 Standards

4.OA.1 Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 × 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.

4.OA.2 Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.

What are students expected to do?

What are the size of numbers that students are expected to work with?

Equations for the Flower Task?

8 x 25 = __

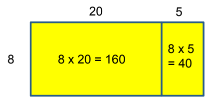
How could students work with this equation?

Place value strategies… (8 x 10) +(8 x 10) + (8 x 5)

More place value… (8 x 20) + (8 x 5)

Familiarity with multiples of 25... (4 x 25) + (4 x 25)

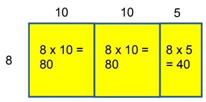
- Which strategy is most common?
- Why?



160 + 40 =200

8 x 20 = 160
8 x 5 = 40
8 x 25 = 200

8 x 20 = 160
8 x 21 = 168
8 x 22 = 176
8 x 23 = 184
8 x 24 = 192
8 x 25 = 200



80 + 80 + 40 =200

Equations for the Flower Task?

8 x 25 = __

Place value… (8 x 10) + (8 x 10) + (8 x 5)
8 x 10 = 80
8 x 20 = 160 -- when I keep one factor constant and double the other factor what happens to the product?
8 x 5 = 40… when I keep one factor constant and half the other factor what happens to the product?
What does the student understand if they use this strategy?

Equations for the Flower Task?

8 x 25 = __
How else might students work with this equation?
What are some other equations that yield the same product?
How are they related to 8 x 25?

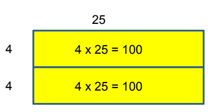
What is happening?

Making an easier problem.. (4 x 25) + (4 x 25)

- This is really 2 groups of 4 times 25, isn't it?
- How can we model this with an equation?

2 x (4 x 25)

2 x (4 x 25)



100 + 100 = 200

Is 2 x (4 x 25) = 4 x 50?

100 + 100 = 200

Equations for the Flower Task?

Is the following true?
25 x 16 = 25 x 8 x 2 = 25 x (8 x 2) = (25 x 2) x 8

If so, why?
Which computation will Grade 4 students gravitate to?

As the meanings and properties of operations develop, students develop computational methods in tandem.

The ***OA Progression*** in Kindergarten and Grade 1 describes this development for single digit addition and subtraction, culminating in methods that rely on properties of operations. The ***NBT Progression*** describes how these methods combine with place value reasoning to extend computation to multi-digit numbers. The ***NF Progression*** describes how the meanings of operations combine with fraction concepts to extend computation to fractions.

Solve this task…

A rectangular prism has a length of 18 inches, a height of 11 inches and a width of 5 inches.

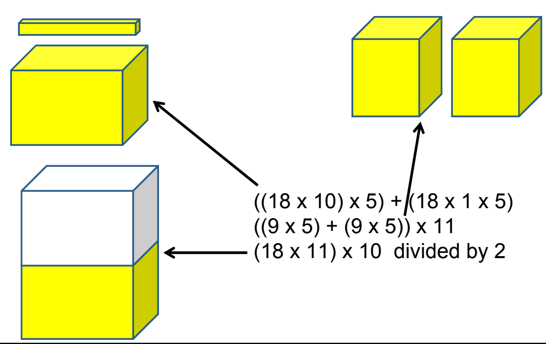
What is the volume of the rectangular prism?

Representations?

Equations?

18 x 11 x 5 …. strategies?

What reasoning underlies each strategy?
(18 x 5) x 11
((18 x 10) x 5) + (18 x 1 x 5)
((9 x 5) + (9 x 5)) x 11
(18 x 11) x 10 divided by 2



Standards

5.MD.5b
Apply the formulas V = l × w × h and V = b × h for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems.

Student Misconceptions

I can make an equivalent problem like this:
54 + 26 = 60 + 20 = 50 + 30 so…..

8 x 12 = 10 x 2 Why or why not?

8 x 12 = 8 x 6 x 2 = 48 x 2 = 16 x 6 WOW!

Extend to rational numbers

33 ⅓ x 12 =

4.5 x 25 =

Take aways?

What are your major takeaways concerning whole number multiplication work?

Doubling and halving:
Is 8 x 25 = 4 x 50?

Associative/ Commutative Property:
Is 4 x 12 x 2 = 8 x 12?

Distributive Property:
Is 15 x 12 = 15 x 10 + 15 x 2?

How to support this work….

- Articulate generalizations implicit in students' work, as well as those students state explicitly, as they notice properties of the operations and relationships among those operations
- Express those generalizations in natural language, in numeric form and with representations.
- Examine how students build on generalizations from one operation to another
- Explore "representation based proof"

Also……

- Listen for students' attempts to articulate claims
- Look for evidence of application of a generalization in their recorded work
- Give students opportunities to notice patterns and them press them to ask why those patterns are there
- Ask: Why is that happening? Will that always be true?
- Create anchor charts of possible claims and charts of justified claims
- Give students plenty of opportunities to use representation based proof to justify their claims

And Finally……

Over time, students build their understanding of the properties of arithmetic: commutativity and associativity of addition and multiplication, and distributivity of multiplication over addition. Initially, they build intuitive understandings of these properties, and ***they use these intuitive understandings in strategies to solve real-world and mathematical problems.*** Later, these understandings become more explicit and allow students to extend operations into the system of rational numbers.

Comments and questions?

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