# K-2 Developing Algebraic Thinking

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## Goals

Explore and discuss tasks related to algebraic reasoning in Grades K-2.

Analyze student responses to tasks

## Driving Questions

What does algebraic thinking look like in Grades K-2?

What types of work should students be doing to develop algebraic thinking?

## Connection to Standards for Math Practice

SMP 2   Reason abstractly and quantitatively

*Quantitative reasoning entails habits of creating a coherent representation of the problem at hand*

For example, when a student sees the expression 40 + 26, she might visualize this problem by thinking, if I have 26 marbles and Marie has 40, how many more do I need to have as many as Marie? Then, in that context, she thinks, 4 more will get me to a total of 30, and then 10 more will get me to 40, so the answer is 14.

## Connection to Math Practice 2

How are students **decontextualizing**?  How do they choose a solution path that may or may not match the structure of the problem and the referents?

How are students **contextualizing** ? How do students wrap numbers in a context? How do they pause during/after computation to probe into the referents?

## Let's explore:

Solve the following using representations and equations.

*Max had 3 blocks. He found some more blocks. Then he had 7 blocks. How many blocks did he find?*

## Max's Blocks

Representations?

Equations?

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## Let's explore:

Turn to a partner. How would a student use counters or cubes to solve this task?

*Max had 3 blocks. He found some more blocks. Then he had 7 blocks. How many blocks did he find?*

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## Max's Blocks

Is there an action and if so, what is the action of the problem?

What are some possible student errors?

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## Zenobia

I went over to work with Zenobia on a story problem because she looked horribly confused.

Zenobia had 3 cubes and 7 cubes and wasn't sure what to do. She counted all of them and got 10, but she looked at me with confusion and said, "I know that's not the answer."

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## Zenobia

How did Zenobia think about the problem?

What teacher moves would support Zenobia in productive struggle?

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## Zenobia's Teacher

Then I wondered if making a connection to something more familiar would help her, so I brought her back to another activity, one at which she is routinely successful.

I said, "Let's put this aside for a moment and solve another problem. Pretend that you and I are playing 'How Many Am I Hiding?' We're playing with 6 cubes. I have some behind my back. You can see 2. You know that there are 6 cubes all together. How many am I hiding?"

Zenobia thought and said, "Four."

I asked, "Does it remind you of anything we've just been doing?"

Zenobia replied, "Not really."

I said, "OK, then let's pretend that Max was playing 'How Many Am I Hiding?' He could see 3. Some were behind his friend's back. He knew the total was 7. How many were hiding?"

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Zenobia thought and then said, "Four."

I asked, "OK, so do you see any connections between this problem and the other Max problem?"

She said, "Yes, there's a 3 and a 7. I just don't get it, still."

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## Let's reflect with the teacher...

For me, the connection between the story problem and the "How Many Am I Hiding?" game seems so obvious.

What is the connection Zenobia needs to make? And what about her classmates?

I am thinking about how to start a class discussion to see what ideas students might have about the connections between the game and the story problems.

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Glossary
Table 1 Common addition and subtraction situations[1]

| | Result Unknown | Change Unknown | Start Unknown |
|---|---|---|---|
| Add to | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ | Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ One-Step Problem |
| Take from | Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ One-Step Problem |

| | Total Unknown | Addend Unknown | Both Addends Unknown[2] |
|---|---|---|---|
| Put Together/ Take Apart[3] | Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ | Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$ | Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$ |

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### Operations and Algebraic Thinking

The Progression in Operations and Algebraic Thinking deals with the basic operations—the kinds of quantitative relationships they model and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships.

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### Let's explore:

Solve the following using representations and equations:

*Bill has some trucks. He gave 7 away. Now Bill has 8 trucks. How many trucks did Bill have?*

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## Bill's Trucks

Representations?

Equations?

## Bill's Trucks

What are some possible student errors?

## Bill's Teacher

When I approached Bill's desk he had a pile of 8 counters total. I asked him, "How are you going to solve the task?" he counted out 7 counters and put them in a pile so he had a pile of 7 and a pile of 1.

## Bill's Teacher

What is Bill's misconception?

What teacher moves would support Bill in productive struggle?

Glossary
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|---|---|---|---|
| Add to | Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ (K) | Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ (1st) | Some bunnies were on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ One-Step Problem (2nd) |
| Take from | Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ (K) | Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ (1st) | Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ One-Step Problem (2nd) |

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| | Difference Unknown | Bigger Unknown | Smaller Unknown |
|---|---|---|---|
| Compare[4] | ("How many more?" version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? (1st) ("How many fewer?" version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5, 5 - 2 = ?$ (1st) One-Step Problem | (Version with "more"): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? (Version with "fewer"): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?, 3 + 2 = ?$ (2nd) One-Step Problem | (Version with "more"): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ $7 + 3 = 5$ (Version with "fewer"): Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? One-Step Problem (1st) |

K: Problem types to be mastered by the end of the Kindergarten year.
1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, First Grade students

## Crayon Puzzles

There are 8 crayons. Some are red and some are blue. How many of each could we have? Find all the possible answers.

## Crayon Puzzles

Representations?

Equations?

## Crayon Puzzles

What is the action of the problem?

I have 8 crayons… *what should the student do?* Some are red and some are blue… *how would students explore this idea?*

## Carol's Candies

Carol had 8 pieces of candy. She had 5 pieces of candy fewer than Steve. How many pieces of candy did Steve have?

## Carol's Candies

Representations?

Actions?

## Carol's Candies

Bobby starts to solve this problem by making a pile of 8 counters. He then says, "fewer means the number goes down." He takes away 5 counters and says, "the answer is 3."

What is Bobby doing?
What teacher moves would support Bobby in productive struggle?

## A progression of comparing...

If I have fewer, what do you have?
If you have more, what do I have?

## Fewer and More

Who has more? How much more?
Ann    Julio
Who has fewer? How much fewer?

## Fewer and More

Maria has 12 cubes.  Ann has more. How many cubes could Ann have?

Ryan has 23 marbles. Mark has fewer. How many marbles could mark have?

## Unpacking a problem...

➔ What is the referent?
➔ What is the language variant?
➔ What relationship is present between quantities?
➔ How might students use the inverse relationship between addition and subtraction to generate a solution strategy?

## Explore these!

Samuel has 5 more pencils than Nancy. Nancy has 6 pencils. How many pencils does Samuel have?

Ike has 4 fewer erasers than Gina. Ike has 5 erasers. How many erasers does Gina have?

## Problem Types Resource

http://cgimathtasks.pbworks.com/

Elemath.pbworks.com

## Tieing it All Together--
### similarities and differences?

❑ Change Unknown
❑ Start Unknown
❑ Both Addends Unknown
❑ Compare- Bigger Unknown/More Version
❑ Compare- Fewer Unknown/ Fewer Version
❑ Compare- Bigger Unknown/Fewer Version
❑ Compare- Fewer Unknown/More Version

Addition and subtraction are the first operations studied.

***Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time.***

Subtraction comes to be understood as reversing the actions involved in addition and as finding an unknown addend.

## Comments and questions?

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