# K-2 Developing Algebraic Thinking

**Drew Polly/ Marta Garcia** 

#### Goals

Explore and discuss tasks related to algebraic reasoning in Grades K-2.

Analyze student responses to tasks

#### **Driving Questions**

What does algebraic thinking look like in Grades K-2?

What types of work should students be doing to develop algebraic thinking?

## Connection to Standards for Math Practice

SMP 2 Reason abstractly and quantitatively *Quantitative reasoning entails habits of creating a coherent representation of the problem at hand* 

For example, when a student sees the expression 40 + 26, she might visualize this problem by thinking, if I have 26 marbles and Marie has 40, how many more do I need to have as many as Marie? Then, in that context, she thinks, 4 more will get me to a total of 30, and then 10 more will get me to 40, so the answer is 14.

#### **Connection to Math Practice 2**

How are students **decontextualizing**? How do they choose a solution path that may or may not match the structure of the problem and the referents?

How are students **contextualizing**? How do students wrap numbers in a context? How do they pause during/after computation to probe into the referents?

#### Let's explore:

Solve the following using representations and equations.

Max had 3 blocks. He found some more blocks. Then he had 7 blocks. How many blocks did he find?



#### Max's Blocks

Representations?

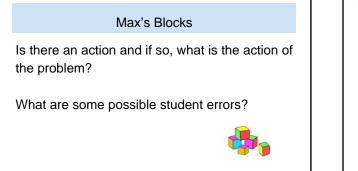
Equations?

#### Let's explore:

Turn to a partner. How would a student use counters or cubes to solve this task?

Max had 3 blocks. He found some more blocks. Then he had 7 blocks. How many blocks did he find?





#### Zenobia

I went over to work with Zenobia on a story problem because she looked horribly confused.

Zenobia had 3 cubes and 7 cubes and wasn't sure what to do. She counted all of them and got 10, but she looked at me with confusion and said, "I know that's not the answer."

#### Zenobia

How did Zenobia think about the problem?

What teacher moves would support Zenobia in productive struggle?

# Zenobia's Teacher

Then I wondered if making a connection to something more familiar would help her, so I brought her back to another activity, one at which she is routinely successful.

I said, "Let's put this aside for a moment and solve another problem. Pretend that you and I are playing 'How Many Am I Hiding?' We're playing with 6 cubes. I have some behind my back. You can see 2. You know that there are 6 cubes all together. How many am I hiding?" Zenobia thought and said, "Four."

I asked, "Does it remind you of anything we've just been doing?"

Zenobia replied, "Not really."

I said, "OK, then let's pretend that Max was playing 'How Many Am I Hiding?' He could see 3. Some were behind his friend's back. He knew the total was 7. How many were hiding?" Zenobia thought and then said, "Four."

I asked, "OK, so do you see any connections between this problem and the other Max problem?"

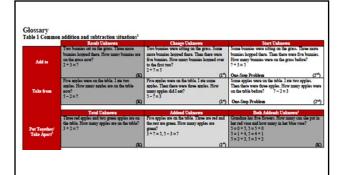
She said, "Yes, there's a 3 and a 7. I just don't get it, still."

#### Let's reflect with the teacher...

For me, the connection between the story problem and the "How Many Am I Hiding?" game seems so obvious.

What is the connection Zenobia needs to make? And what about her classmates?

I am thinking about how to start a class discussion to see what ideas students might have about the connections between the game and the story problems.



#### **Operations and Algebraic Thinking**

The Progression in Operations and Algebraic Thinking deals with the basic operations—the kinds of quantitative relationships they model and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships.

#### Let's explore:

Solve the following using representations and equations:

Bill has some trucks. He gave 7 away. Now Bill has 8 trucks. How many trucks did Bill have?



# **Bill's Trucks**

Representations?

Equations?

#### **Bill's Trucks**

What are some possible student errors?



# **Bill's Teacher**

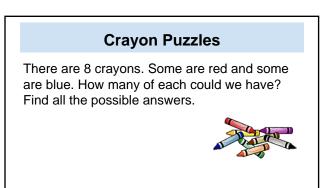
When I approached Bill's desk he had a pile of 8 counters total. I asked him, "How are you going to solve the task?" he counted out 7 counters and put them in a pile so he had a pile of 7 and a pile of 1.

# **Bill's Teacher**

What is Bill's misconception?

What teacher moves would support Bill in productive struggle?

	Result Unknown	Change Unknown	Start Unknown
	Two bunnies sat on the grass. Three more	Two bunnies were sitting on the grass. Some	Some bunnies were sitting on the grass. Three more
	bunnies hopped there. How many bunnies are	more bunnies hopped there. Then there were	burnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?
Add to	on the grass now? 2+3=2	to the first tan?	How many bunnes were on the grass before? 2 + 3 = 1
		2+7=5	
	(X)	(1)	One-Step Problem (2 <sup>24</sup> )
Tale from	Five apples were on the table. I ate two amples. How many apples are on the table	Five apples were on the table. I ate some apples. Then there were three apples. How	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were
	apples. How many apples are on the table	apples. Then there were three apples. How many apples did I est?	Then there were three apples. How many apples were on the table before? 2 - 2 = 3
	5-2=2	5-7=3	
	60	an)	One-Step Problem (2 <sup>nd</sup> )
	Total Unknown	Addend Unknown	Both Addends Unknown <sup>4</sup>
	Three red apples and two green apples are on the table. How many apples are on the table?	Five apples are on the table. Three are red and the rest are green. How many apples are	Grandma has five flowers. How many can she put m her red vase and how many in her blue vase?
Put Torether/	3+2=7	meren?	5 = 0 + 5, $5 = 5 + 0$
Take Apart		3+7=5.5-3=7	5=1+4.5=4+1
			5=2+3,5=3+2
	(A)	( <sup>1</sup> )	(60)
	Difference Unknown	Birrer Unknown	Smaller Unknown
	("How many more?" version):	(Version with "more"):	(Version with "more"):
	Lucy has two apples. Julie has five apples.	Julie has three more apples than Lucy, Lucy	Julie has 3 more apples than Lucy. Julie has five
Compare	How many more apples does Julie have than	has two apples. How many apples does Julie	apples. How many apples does Lucy have?
	Lucy?	have?	
		One-Step Problem (1")	5-3=7 7+3=5
	("How many ferrer?" version):	One-Step Problem (1") (Version with "feaver"):	One-Step Problem (2 <sup>nd</sup> ) (Varsion with "Sawar"):
	("How many lewer?" version): Lucy has two apples. Julie has five apples.	(Version with "lower"): Lucy has 3 fewer apples than Julie, Lucy has	(Version with "hewer"): Lucy has three fewer apples than Julie. Julie has five
	How many fewer apples. June has new apples.	two apples. How many apples does Julie have?	amles. How many amles does Lucy have?
	Julia?	2+3=7,3+2=7	and the second sec
	2+7=5,5-2=7		
		One-Step Problem (2 <sup>nd</sup> )	One-Step Problem (1"



# **Crayon Puzzles**

Representations?

Equations?

# **Crayon Puzzles**

What is the action of the problem?

I have 8 crayons... what should the student do? Some are red and some are blue... how would students explore this idea?

# **Carol's Candies**

Carol had 8 pieces of candy. She had 5 pieces of candy fewer than Steve. How many pieces of candy did Steve have?



#### **Carol's Candies**

Representations?

Actions?

# **Carol's Candies**

Bobby starts to solve this problem by making a pile of 8 counters. He then says, "fewer means the number goes down." He takes away 5 counters and says, "the answer is 3."

What is Bobby doing? What teacher moves would support Bobby in productive struggle?

# A progression of comparing... If I have fewer, what do you have? If you have more, what do I have?



#### **Fewer and More**

Who has more? How much more? Ann Julio Who has newer? How much fewer?



#### **Fewer and More**

Maria has 12 cubes. Ann has more. How many cubes could Ann have?

Ryan has 23 marbles. Mark has fewer. How many marbles could mark have?



# Unpacking a problem...

- →What is the referent?
- → What is the language variant?
- → What relationship is present between quantities?
- How might students use the inverse relationship between addition and subtraction to generate a solution strategy?

#### **Explore these!**

Samuel has 5 more pencils than Nancy. Nancy has 6 pencils. How many pencils does Samuel have?

Ike has 4 fewer erasers than Gina. Ike has 5 erasers. How many erasers does Gina have?

Advert         Instantia hopped faces. Then there were fixed in the face were fixed were fixed in the face were fixed were f		Result Unknown	Change Unknown	Start Unknown
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$ \begin{array}{c} \text{Add} & 2 + 2 + 2 \\  First series was not set that the link. It is not set that the link of the link of$				bunnies hopped there. Then there were five bunnies.
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The value was in the table. To the table of		2+3=?		?+3=5
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The two spin for unrespin to a the shift is a proving the state of the spin transformation of the spi			Five apples were on the table. I ate some	Some apples were on the table. I ate two apples
Table one         and starting product (1 and 2 - 2 - 7)         and product (2 - 2 - 3)         and product (2 - 3)         an			apples. Then there were three apples. How	Then there were three apples. How many apples were
Comparing a first space of the space	Take from		many apples did I eat?	on the table before? ?-2=3
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The Superior Table 2015 and the set of the		Total Unknown	Addend Takasara	Both Addards Unknown <sup>1</sup>
Description         Description respective set of the soft as profile are green. Here are profile are soft as profile are profile				Grandma has five flowers. How many can she put in
$\frac{1}{2} + \frac{1}{2} + \frac{1}$		the table. How many apples are on the table?	the rest are green. How many apples are	her red vase and how many in her blue vase?
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Put Together/	3+2=7	green?	
$\begin{array}{c c c c c c c c c c c c c c c c c c c $	Take Apart		3+7=5,5-3=7	
$ \begin{array}{c} (Her many more') variable \\ Leych her weight, Shink her we$				3=2+3,3=3+2
$ \begin{array}{c} (Her many more') variable \\ Leych her weight, Shink her we$		(A)	(A)	(%)
Lear has two series, John has the weights product array many spin do An John has the main large structure of the series of th			Birrer Unknown	Smaller Unknown
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Compart         (f)         Data Sing Tranking         Data Sing Tranking <thdata sing="" th="" tranking<="">         Data Sing Tranki</thdata>	Compare <sup>4</sup>	How many more apples does Julie have than	has two apples. How many apples does Julie	apples. How many apples does Lucy have?
Compart         (?)         One-Stop Problem         (?)         Dates that the stop Problem         (?)         Dates the problem         Dates the problem         (?)         Dates the problem         (?)         Dates the problem         Dates the problem <thdates p<="" td="" the=""><td>Lacy?</td><td>have?</td><td></td></thdates>		Lacy?	have?	
(How many force? variant): Lacy has to regreps. In his fast for apple. How many force apples does Lacy have than his?		65	One Sum Daublant (15)	
Lucy has two apples. Julia has five apples. How many fewer apples does Lucy have than Julia? Julia?				
How many fewer apples does Lucy have than two apples. How many apples does Julie have? apples. How many apples does Lucy have? Julie?				Lucy has three fewer areas than Julie. Julie has five
		How many fewer apples does Lucy have than	two apples. How many apples does Julie have?	
(1 <sup>th</sup> ) One-Step Problem (2 <sup>th</sup> ) One-Step Problem				

# **Problem Types Resource**

http://cgimathtasks.pbworks.com/

Elemath.pbworks.com

# Tieing it All Together-similarities and differences?

Change Unknown

- Start Unknown
- Both Addends Unknown
- Compare- Bigger Unknown/More Version
- Compare- Fewer Unknown/ Fewer Version
- Compare- Bigger Unknown/Fewer Version
- Compare- Fewer Unknown/More Version

Addition and subtraction are the first operations studied.

Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time.

Subtraction comes to be understood as reversing the actions involved in addition and as finding an unknown addend.

# **Comments and questions?**

Marta: martagarcia0901@gmail.com

Drew: drew.polly@uncc.edu

