Bar Diagrams for Operations - *enVisionMATH*

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Bar Diagrams for Addition and Subtraction Situations

| Problem Type Joining | Example A Total Amount Unknown | Example B Amount Joined Unknown | Example C Initial Amount Unknown |
|--|---|---|---|
| Ü | Kim has 23 antique dolls. Her Father gives her 18 more antique dolls. Now how many antique dolls does she have? | Debbie has saved \$57. How much more money does she need in order to have \$112? | Tom had some money in his savings account. He then deposited \$45 into the same account. Then he had \$92 in all. How much did he have in his savings account to start? |
| Diagram Showing the Relationship | 23 18 | 57 ? | ? 45 |
| Description of the Relationship | The two unequal amounts (23 and 18) are known and being joined and the total is unknown. | The initial amount is known (57). The amount being joined to that is unknown. The total is known (112). | The initial amount is unknown. The amount being joined to that is known (45) and the total is known (92). |
| Number Sentence | 23 + 18 = ? | 57 + ? = 112 112 - 57 = ? | ? + 45 = 92 92 - 45 = ? |
| Problem Type | Example D Amount Remaining Unknown | Example E Amount Separated | Example F Initial Amount Unknown |

| Problem Type Separating | Example D Amount Remaining Unknown | Example E Amount Separated Unknown | Example F Initial Amount Unknown |
|--|--|---|---|
| | Steven has 122 jelly beans. He eats 71 of them in one weekend. How many jelly beans are left? | Carrie has 45 CDs. She gives some to Jo. Now Carrie has 27 left. How many did she give to Jo? | Alan has some marbles. He lost 12 of them. Then he had 32 left. How many did he have before he lost some? |
| Diagram Showing the Relationship | 71 ? | ? 45 | |
| Description of the Relationship | The total amount is known (122) and the amount separated from that is known (71). The amount remaining is unknown. | The total amount is known (45) and the amount separated from that is unknown. The amount remaining is known (27). | The total is unknown. The amount separated from the total is known (12) and the amount remaining is known (32). |
| Number Sentence | 122 – 71 = ? | 45 - ? = 27 27 + ? = 45 | ? - 12 = 32 12 + 32 = ? |

Bar Diagrams for Addition and Subtraction Situations

| Problem Type | Example G Whole Unknown | Example H One Part Unknown | Example I Another Part Unknown |
|--|--|--|---|
| Part-Part- Whole | Fourteen cats and 16 dogs are in the kennel. How many dogs and cats are in the kennel? | Some adults and 12 children were on a bus. There are 31 people in all on the bus. How many adults were on the bus? | Forty-nine people went on a hike. Six were adults and the rest were children. How many children went on the hike? |
| Diagram Showing the Relationship | | ? 12 | 6 ? |
| Description of the Relationship | Each unequal part is known (14 and 16); the whole is unknown. | The first part is unknown, but the second part is known (12). The whole is known (31). | The whole is known (49) and the initial part is known (6). The other part is unknown. |
| Number Sentence | 14 + 16 = ? | ? + 12 = 31 31 - 12 = ? | 6 + ? = 49 49 - 6 = ? |

| Problem Type Comparison | Example J Amount More (or Less) Unknown | Example K Smaller Amount Unknown | Example L Larger Amount Unknown |
|------------------------------------|--|---|---|
| | Alex has 47 toy cars. Keisha has 12 cars. How many more cars does Alex have? | Fran spent \$84 which was \$26 more than Alice spent. How much did Alice spend? | Barney has 23 old coins. Steve has 16 more old coins than Barney. How many old coins does Steve have? |
| Diagram Showing the Relationship | 12 ? | ? 26 | 23 16 |
| Description of the Relationship | Two known amounts (47 and 12) are being compared. The amount more/less is unknown. | The larger amount is known (84), and smaller amount is unknown. The amount more the larger is than the smaller is known (26). | One smaller amount is known (23), and the larger amount is not known. The amount more the larger is than the smaller is known (16). |
| Number Sentence | 47 – 12 = ? | 84 - ? = 26 84 - 26 = ? | 23 + 16 = ? ? - 23 = 16 |

$\label{eq:BarDiagrams} \textbf{Bar Diagrams for Multiplication and Division Situations}$

| | Example M Total Amount Unknown | Example N Amount per Group Unknown | Example O Number of Groups Unknown |
|---|--|--|---|
| Problem Type Joining Equal Groups | Kim has 4 photo albums. Each album has 85 pictures. How many photos are in her 4 albums? | Pam had 4 bags and put the same number of apples in each bag. She ended up with 52 apples in bags. How many did she put in each bag? | Fred bought some books that each cost \$16. He spent \$80 altogether. How many books did he buy? |
| Diagram Showing the Relationship | 85 85 85 85 85 85 85 | ? ? ? ? | 16 |
| Description of the Relationship | Four equal known amounts (85) are being joined to find the unknown total. | A known number (4) of unknown but equal amounts are being joined to give a known total (52). | A known amount (16) is being joined an unknown number of times to itself to get a known total (80). |
| Number Sentence | 4 x 85 = ? | 4 x ? = 52 52 ÷ 4 = ? | ? $ x 16 = 80 $ $80 \div 16 = ? $ |

| Problem Type Separating | Example P Amount per Group Unknown | Example Q Number of Groups Unknown | Example R Total Amount Unknown |
|-------------------------------------|--|---|---|
| Equal Groups | Byron has 45 pigeons. He keeps them in 5 pens with the same number of pigeons in each. How many pigeons are in each pen? | A total of 108 children signed up for soccer. How many 18- person teams can be made? | Kim had some cards. She put them into piles of 35 and was able to make 4 piles. How many cards did she have to start? |
| Diagram Showing the Relationship | 7 7 7 7 7 | 18 | 35 35 35 35 |
| Description of the Relationship | The total is known (45) and being separated into a known number of equal groups (5) but the amount in each group is unknown. | The total is known (108) and being separated into equal groups of a known amount (18). The number of equal groups needed to match the total is unknown. | The total amount is unknown. It is separated into a known number of groups (4) with a known equal amount in each (35). |
| Number Sentence | $45 \div 5 = ?$ | 216 ÷ 18 = ? 18 x ? = 216 | ? ÷ 4 = 35 4 x 35 = ? |

Bar Diagrams for Multiplication and Division Situations

| Problem Type | Example S Larger Amount Unknown | Example T Smaller Amount Unknown | Example U Number of Times as Many Unknown |
|-------------------------------------|---|--|--|
| Comparison | Alex has 17 toy cars. Keisha has 3 times as many. How many cars does Keisha have? | Barney has 24 old coins. This is 3 times more coins than Steve has. How many old coins does Steve have? | Ann's teacher is 39 years old. Ann is 13 years old. Ann's teacher is how many times as old as Ann? |
| Diagram Showing the Relationship | ? 17 17 17 3 times as many | ? ? ? 3 times as many | 39 13 |
| Description of the Relationship | The smaller amount is known (17) and the larger amount is a given number of times more (3). The larger quantity is not known. | The larger amount is known (24) and is a given number of times greater than the small amount (3). The smaller amount is not known. | The larger amount (39) and the smaller amount (13) are known. How many times more the larger amount is than the smaller amount is not known. |
| Number Sentence | 3 x 17 = ? | $3 \times ? = 24$ $24 \div 3 = ?$ | ? $\times 13 = 39$ 39 ÷ 13 = ? |