

## Addition and Subtraction Problem Types

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## Credit goes to work from...

- NCDPI
- Cognitively Guided Instruction researchers and authors
- Colleagues including but not limited to Marta Garcia
- Tools for teachers MSP Grant

## Let's do some math...

- There are some students in the cafeteria. Then 18 more students arrive. Ten minutes later a total of 37 students leave. If there are now 56 children in all in the cafeteria how many were first there?
- Show your work in 2 ways and write an equation that helped you solve the problem.
- Look at your equation- does it match the context of the problem?

## Debriefing Cafeteria Task

- Rigorous?
  - If so, why?
- Common incorrect answers?
  - Why?
- Strategies to support students solving of this problem...

## Key words vs. action

- There are some students in the cafeteria. Then 18 more students arrive. Ten minutes later a total of 37 students leave. If there are now 56 children in all in the cafeteria how many were first there?
- What do these words or phrases mean? More, Total, In All
- What does the action of arriving and leaving tell you compared to key words?

## Equations to Solve vs. Equations that Match

- There are some students in the cafeteria. Then 18 more students arrive. Ten minutes later a total of 37 students leave. If there are now 56 children in all in the cafeteria how many were first there?

$$56 + 37 - 18 = \underline{75} \quad \text{OR} \quad \underline{75} + 18 - 37 = 56$$

- Does the equation writing matter?

## Standards for Mathematical Practice

- SMP 2- Reason abstractly and quantitatively.
- Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects.
- When children leave .... When children arrive .....

## Problem Types

- What do you know about them?
- Where do students struggle?
- What strategies do you think work?

## Goals

- Analyze tasks to identify problem type
- Make connections between tasks, representations, and equations
- Consider *high-leverage* practices to support students
- Look at student work related to problem types

	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$ (K)	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$ (1 <sup>+</sup> )	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$ One-Step Problem (2 <sup>+</sup> )
Take from	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$ (K)	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$ (1 <sup>+</sup> )	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$ One-Step Problem (2 <sup>+</sup> )
Put Together/ Take Apart <sup>2</sup>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$ (K)	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ , $5 - 3 = ?$ (1 <sup>+</sup> )	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 - 2 = 3$ , $5 - 5 = 0$ $5 = 1 + 4$ , $5 = 4 + 1$ $5 = 2 + 3$ , $5 = 3 + 2$ (K)
Compare <sup>3</sup>	Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? ("How many more?" version) Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ , $5 - 2 = ?$ (1 <sup>+</sup> )	Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? ("How many fewer?" version) Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ , $3 + 2 = ?$ (1 <sup>+</sup> )	Julie has 3 more apples than Lucy. Julie has five apples. How many apples does Lucy have? ("How many fewer?" version) Lucy has three fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$ One-Step Problem (2 <sup>+</sup> )

## Let's explore

Solve the following using representations and equations.

*Max had 3 blocks. He found some more blocks. Then he had 7 blocks. How many blocks did he find?*

## Max's Blocks

Representations?

Equations?

## Max's Blocks

Is there an action and if so, what is the action of the problem?

What are some possible student errors?



## Let's explore

Turn to a partner. How would a student use counters or cubes to solve this task?

*Max had 3 blocks. He found some more blocks. Then he had 7 blocks. How many blocks did he find?*



## Let's explore

Solve the following using representations and equations.

*Monique had some blocks. She found 3 more blocks. Now she had 7 blocks. How many blocks did she start with?*

## Zenobia

I went over to work with Zenobia on a story problem because she looked horribly confused.

Zenobia had 3 cubes and 7 cubes and wasn't sure what to do. She counted all of them and got 10, but she looked at me with confusion and said, "I know that's not the answer."

## Zenobia

How did Zenobia think about the problem?

What teacher moves would support Zenobia in productive struggle?

## Zenobia's Teacher

Then I wondered if making a connection to something more familiar would help her, so I brought her back to another activity, one at which she is routinely successful. I said, "Let's put this aside for a moment and solve another problem. Pretend that you and I are playing 'How Many Am I Hiding?' We're playing with 6 cubes. I have some behind my back. You can see 2. You know that there are 6 cubes all together. How many am I hiding?"

Zenobia thought and said, “Four.”

I asked, “Does it remind you of anything we’ve just been doing?”

Zenobia replied, “Not really.”

I said, “OK, then let’s pretend that Max was playing ‘How Many Am I Hiding?’ He could see 3. Some were behind his friend’s back. He knew the total was 7. How many were hiding?”

Zenobia thought and then said, “Four.”

I asked, “OK, so do you see any connections between this problem and the other Max problem?”

She said, “Yes, there’s a 3 and a 7. I just don’t get it, still.”

Let’s reflect with the teacher...

For me, the connection between the story problem and the “How Many Am I Hiding?” game seems so obvious.

What is the connection Zenobia needs to make? And what about her classmates?

I am thinking about how to start a class discussion to see what ideas students might have about the connections between the game and the story problems.

**Glossary**  
Table 1 Common addition and subtraction situations<sup>1</sup>

	Ready Unknown	Change Unknown	Start Unknown
<b>Add to</b>	Two bunions sat on the grass. Three more bunions hopped there. How many bunions are on the grass now? $2 + 3 = ?$	Two bunions were sitting on the grass. Some more bunions hopped there. Then there were five bunions. How many bunions hopped over to the first two? $2 + ? = 5$	Some bunions were sitting on the grass. Three more bunions hopped there. Then there were five bunions. How many bunions were on the grass before? $? + 3 = 5$
<b>Take from</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
<b>Put Together/ Take Apart</b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5, 5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5, 5 = 5 + 0$ $5 = 1 + 4, 5 = 4 + 1$ $5 = 2 + 3, 5 = 3 + 2$

## Operations and Algebraic Thinking

The Progression in Operations and Algebraic Thinking deals with the basic operations—the kinds of quantitative relationships they model and consequently the kinds of problems they can be used to solve as well as their mathematical properties and relationships.

## Let’s explore:

Solve the following using representations and equations:

*Bill has some trucks. He gave 7 away. Now Bill has 8 trucks.  
How many trucks did Bill have?*



### Bill's Trucks

Representations?

Equations?



### Bill's Trucks

What are some possible student errors?

### Bill's Teacher

When I approached Bill's desk he had a pile of 8 counters total. I asked him, "How are you going to solve the task?" he counted out 7 counters and put them in a pile so he had a pile of 7 and a pile of 1.

### Bill's Teacher

What is Bill's misconception?

What teacher moves would support Bill in productive struggle?

### Carol's Candies

Carol had 8 pieces of candy. She had 5 pieces of candy fewer than Steve. How many pieces of candy did Steve have?



### Carol's Candies

Representations?

Actions?

### Carol's Candies

Bobby starts to solve this problem by making a pile of 8 counters. He then says, "fewer means the number goes down." He takes away 5 counters and says, "the answer is 3."

What is Bobby doing?

What teacher moves would support Bobby in productive struggle?

### A progression of comparing...

If I have fewer, what do you have?

If you have more, what do I have?



### Fewer and More

Who has more? How much more?

Ann



Julio

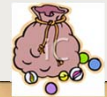


Who has fewer? How much fewer?

### Fewer and More

Maria has 12 cubes. Ann has more. How many cubes could Ann have?

Ryan has 23 marbles. Mark has fewer. How many marbles could Mark have?



### Unpacking a problem...

- What do we know?
- What is the language related to comparing?
- What relationship is present between quantities?
- How might students use the relationship between addition and subtraction to generate a solution strategy?

### Explore these!

Samuel has 5 more pencils than Nancy. Nancy has 6 pencils. How many pencils does Samuel have?

Ike has 4 fewer erasers than Gina. Ike has 5 erasers. How many erasers does Gina have?

**Glossary**  
**Table 1 Common addition and subtraction situations<sup>1</sup>**

	Join Unknown	Change Unknown	Start Unknown
<b>Add to</b>	Two buzzards sat on the grass. Three more buzzards hopped there. How many buzzards are on the grass now? $2 + 3 = ?$	Two buzzards were sitting on the grass. Some buzzards hopped there. Then there were five buzzards. How many buzzards hopped over to the first two? $2 + ? = 5$	Some buzzards were sitting on the grass. Three more buzzards hopped there. Then there were five buzzards. How many buzzards were on the grass before? $? + 3 = 5$
<b>Take from</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
<b>Put Together/ Take Apart<sup>2</sup></b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Two are red and the rest are green. How many apples are green? $5 - 2 = ?$ , $5 - 3 = ?$	Grandma left five flowers. Grandma gave two put in her red vase and how many in her blue vase? $5 - 2 = ?$ , $5 - 3 = ?$ $2 + 1 = 3$ , $3 + 2 = 5$
<b>Compare<sup>3</sup></b>	["How many more?" version] Lucy has two apples. Julia has five apples. How many more apples does Julia have than Lucy? $5 - 2 = ?$	["Version with 'more'"] Julia has three more apples than Lucy. Lucy has two apples. How many apples does Julia have? $2 + 3 = ?$	["Version with 'more'"] Julia has 3 more apples than Lucy. Julia has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$
	["How many fewer?" version] Lucy has two apples. Julia has five apples. How many fewer apples does Lucy have than Julia? $5 - 2 = ?$ , $5 - 3 = ?$	["Version with 'fewer'"] Lucy has 3 fewer apples than Julia. Lucy has two apples. How many apples does Julia have? $2 + 3 = ?$ , $5 - 3 = ?$	["Version with 'fewer'"] Lucy has three fewer apples than Julia. Julia has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$

1. Problem types to be mastered by the end of the Kindergarten year.  
2. Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, First Grade students

## Problem Types Resource

<http://cgimathtasks.pbworks.com/>

<http://elemath.pbworks.com>

## Tying it All Together-- similarities and differences?

- ☐ Change Unknown
- ☐ Start Unknown
- ☐ Both Addends Unknown
- ☐ Compare- Bigger Unknown/More Version
- ☐ Compare- Fewer Unknown/Fewer Version
- ☐ Compare- Bigger Unknown/Fewer Version
- ☐ Compare- Fewer Unknown/More Version

Addition and subtraction are the first operations studied.

*Initially, the meaning of addition is separate from the meaning of subtraction, and students build relationships between addition and subtraction over time.*

Subtraction comes to be understood as reversing the actions involved in addition and as finding an unknown addend.

## What we know from research...

- Not all tasks are created equal
- Rigor increases as we move from the left to the middle to the right
- Grade level expectations are explicit (since Common Core)

	Join Unknown	Change Unknown	Start Unknown
<b>Add to</b>	Two buzzards sat on the grass. Three more buzzards hopped there. How many buzzards are on the grass now? $2 + 3 = ?$	Two buzzards were sitting on the grass. Some buzzards hopped there. Then there were five buzzards. How many buzzards hopped over to the first two? $2 + ? = 5$	Some buzzards were sitting on the grass. Three more buzzards hopped there. Then there were five buzzards. How many buzzards were on the grass before? $? + 3 = 5$
<b>Take from</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
<b>Put Together/ Take Apart<sup>2</sup></b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Two are red and the rest are green. How many apples are green? $5 - 2 = ?$ , $5 - 3 = ?$	Grandma left five flowers. Grandma gave two put in her red vase and how many in her blue vase? $5 - 2 = ?$ , $5 - 3 = ?$ $2 + 1 = 3$ , $3 + 2 = 5$
<b>Compare<sup>3</sup></b>	["How many more?" version] Lucy has two apples. Julia has five apples. How many more apples does Julia have than Lucy? $5 - 2 = ?$	["Version with 'more'"] Julia has three more apples than Lucy. Lucy has two apples. How many apples does Julia have? $2 + 3 = ?$	["Version with 'more'"] Julia has 3 more apples than Lucy. Julia has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$
	["How many fewer?" version] Lucy has two apples. Julia has five apples. How many fewer apples does Lucy have than Julia? $5 - 2 = ?$ , $5 - 3 = ?$	["Version with 'fewer'"] Lucy has 3 fewer apples than Julia. Lucy has two apples. How many apples does Julia have? $2 + 3 = ?$ , $5 - 3 = ?$	["Version with 'fewer'"] Lucy has three fewer apples than Julia. Julia has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$

## What we also know from research...

- Strategies vary widely
- Strategies that are appropriate may include things that look like addition or subtraction
- Algebraic reasoning is thoroughly embedded in these tasks and possible discussion points

### Resources for New Standards

- Tools for Teachers Project
  - Develop PD (6 full days) for K-5 teachers on mathematics teaching and learning with focus on new mathematics standards
  - Collaborate with NCDPI to update and create resources (NC Lessons for Learning, Formative assessment tasks, Parent letters, other resources)
- NC Math Collaborative
  - Putting together pacing on new math standards
  - Contact is Katie Schwartz (East Carolina)

### Continue your learning...

- State-wide math add-on license
  - UNC Charlotte, 100% online with not online meetings
  - NC State, face-to-face cohorts
  - UNC/ECU/UNC-W- 100% online with some online meetings
- NCCTM- Greensboro in early November
- Drew.Polly@unc.edu
- <http://elemath.pbworks.com>

### Evaluation

- <http://tinyurl.com/2017MathSummit>
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